Hyponormal measurable operators, affiliated to a semifinite von Neumann algebra

Abstract :

Let M be a von Neumann algebra of operators on a Hilbert space H and τ be a faithful normal semifinite trace on M. Denote by $S(\mathrm{M},\tau)$ the *-algebra of all τ-measurable operators.

Let $U \in \mathrm{M}$ be an isometry and $P \in \mathrm{M}$ be a projection.

If $UPU^* \leq P$ and $\tau(P) < +\infty$, then $UP = PU$.

If $A \in S(\mathrm{M},\tau)$ is hyponormal and $U \in \mathrm{M}$ is an isometry such that $A^* = UAU^*$, then the operator A is normal.

Let an operator $X \in S(\mathrm{M},\tau)$ be

1. hyponormal, or

2. of the form $X = \lambda I + A$ with some $\lambda \in \mathrm{C}$ and a τ-compact operator $A \in S(\mathrm{M},\tau)$.

If $(\mathrm{Re}X)_+ = |X|$ (or $(\mathrm{Im}X)_+ = |X|$), then $X = |X| \geq 0$.

If the operator $A \in \mathrm{M}$ is normal, $B \in L_1(\mathrm{M},\tau) \cap L_2(\mathrm{M},\tau)$ and the operator $T = A + B$ is hyponormal, then T is normal.

For the operator $A \in L_2(\mathrm{M},\tau)$, the following conditions are equivalent:

(i) A is normal;

(ii) $\tau(PA^*AP) \geq \tau(PAA^*P)$ for all projections $P \in \mathrm{M}$; (iii) $\tau(PA^*AP) \leq \tau(PAA^*P)$ for all projections $P \in \mathrm{M}$.

The operator $A \in \mathrm{M}$ is hyponormal if and only if $\tau(PA^*AP) \geq \tau(PAA^*P)$ for all projections $P \in \mathrm{M}$

with finite $\tau(P)$.

If projections $P,Q \in \mathrm{B(H)}$ and $n \in \mathrm{N}$, then the operator $(PQ)^n$ is hyponormal if and only if $PQ = QP$.

Operator inequalities for degrees of hyponormal contractions are obtained. It is shown that any degree U^n of a hyponormal partial isometry U is a hyponormal partial isometry and $U^{*n}U^n = U^*U$.