Continuous Mappings on Directed Graphs: Definitions, Theorems, and Applications

Abstract :

This work introduces a novel mathematical framework for the study of structurepreserving transformations on directed graphs. We define a mapping $f : V \to V$ on a directed graph $G = (V,E)$ to be *continuous* if the image of every simple directed path is itself a simple directed path or a single vertex. This definition provides a discrete analogue of topological continuity, ensuring that the fundamental path structure of the graph is preserved or collapsed in a controlled manner. We demonstrate that the set of all such continuous mappings forms a monoid under function composition and establish key theorems regarding the preservation of graph properties like connectivity. We then introduce the concept of an *Ncompressor* for multiscale graph compression, proving the existence of optimal compressors. The significance of this framework is highlighted through its potential applications in simplifying and analyzing complex networks in systems biology, social sciences, and computer science. Finally, we provide key references for further reading.